

Experimental observation of the ‘Tilting Mode’ of an array of vortices in a dilute Bose-Einstein Condensate

N. L. Smith, W. H. Heathcote, J. M. Krueger and C. J. Foot
*Clarendon Laboratory, Department of Physics, University of Oxford,
 Parks Road, Oxford, OX1 3PU,
 United Kingdom.*

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We have measured the precession frequency of a vortex lattice in a Bose-Einstein condensate of Rb^{87} atoms. The observed mode corresponds to a collective motion in which all the vortices in the array are tilted by a small angle with respect to the z -axis (the symmetry axis of the trapping potential) and synchronously rotate about this axis. This motion corresponds to excitation of a Kelvin wave along the core of each vortex and we have verified that it has the handedness expected for such helical waves, i.e. precession in the opposite sense to the rotational flow around the vortices. The experimental method used to excite this collective mode closely resembles that used to study the scissors mode and excitation of the scissors mode for a condensate containing a vortex array was used to determine the angular momentum of the system. Indeed, the collective tilting of the array that we have observed has previously been referred to as an ‘anomalous’ scissors mode.

Since a vortex was first nucleated in a dilute Bose-condensed gas [1], there has been a considerable effort to understand the dynamical behaviour of individual vortices and vortex arrays. This research has, in part, been driven by the parallels between dilute gas systems and more complex superfluids such as He^4 , however with the Bose-condensed gases it has proved to be straightforward to obtain images of vortices and to measure properties of a single vortex. The precession of a single vortex has been investigated theoretically [2, 3], and studied experimentally in a nearly spherical Bose-condensate [4]. It is also possible to nucleate many vortices in a Bose-condensed gas and these form a regular Abrikosov lattice [5, 6, 7]. The work described here can be described in two complementary ways: (a) as an extension of the previous work on the precession of a single vortex to the case of an array of vortices in an anisotropic trap where the collective motion of the vortices is relatively rapid, or (b) as the excitation of the lowest-energy Kelvin wave of a vortex lattice. This second viewpoint is described in more detail below.

Vortices break the degeneracy of certain modes in the normal Bogoliubov excitation spectrum for a trapped condensate. This splitting has been observed for both the $m=2$ quadrupole mode [8] and also for the scissors mode [9]; in the latter case the precession that arises when the condensate has some angular momentum leads to a ‘superfluid gyroscopic’ motion [10]. More recently an excitation of a vortex in the form of a helical Kelvin wave has been detected [11]; collective oscillations of a vortex lattice called Tkachenko modes have been observed and their frequency measured [12]. In a recent theoretical paper Chevy and Stringari [13] have extended the hydrodynamic theory of a Bose-condensed gas to describe an array of vortices and they predict that an array precesses in a similar way to a single vortex, i.e. all the vortices simultaneously undergo a motion equivalent to the lowest-

energy Kelvin wave of a single vortex (see Fig. 1). (In these waves the orientation of the vortex core changes and, unlike the Tkachenko modes, the Kelvin waves do not arise in a two-dimensional system.) In this paper we present measurements of the frequency of this ‘collective tilting’ mode of the vortex array by an experimental method similar to that used to study the scissors mode of the condensate [14]. Indeed Chevy and Stringari [13] refer to the collective tilting of the array as an ‘anomalous’ scissors mode. Their hydrodynamic theory predicts that the frequencies of this mode and the two normal scissors modes, in a reference frame rotating at frequency Ω_0 , are given by the roots of the cubic equation:

$$\omega^3 \pm 2\Omega_0\omega^2 - \omega(\omega_z^2 + \omega_\perp^2 - \Omega_0^2) \mp 2\Omega_0\omega_z^2 = 0, \quad (1)$$

where ω_\perp and ω_z are the radial and axial angular oscillation frequencies. $\Omega_0 = n_v h/2m$ is the effective rotation frequency of the condensate; n_v is the number of vortices per unit area and m is the mass of the condensed isotope. The hydrodynamic theory assumes a uniform distribution of vortices within the condensate so that it mimics the rotation of a rigid body.

We denote the (angular) frequencies of the upper and lower scissors modes as ω_u and ω_l respectively and the frequency of the tilting mode by ω_t . These frequencies are calculated in the non-rotating, laboratory reference frame. Generally we shall express frequencies in terms of their fraction of the radial frequency of the trap. The three solutions of Eq. 1 in a potential with $\omega_z/\omega_\perp = \sqrt{8}$ are shown in Fig 2 (a). This shows that when $\Omega_0 = 0$ the scissors mode frequency is three times the radial trap frequency, and that the splitting between the upper and lower scissors modes is about $2\Omega_0 \simeq 2\omega_t$. Fig. 2 (b) shows a plot of the three quantities $\omega_u/\omega_\perp - 3$, $3 - \omega_l/\omega_\perp$, ω_t/ω_\perp and also the ratio $(\omega_u - \omega_l)/2\omega_t$. This shows that the frequencies of all the modes vary approximately linearly with Ω_0 (angular momentum); in particular ω_u is very

close to linear over the entire range but both the lower scissors mode and the tilting mode show a noticeable deviation. The given ratio has a value within 15% of unity throughout the range. Note that the frequency of the tilting mode tends to zero as Ω_0 tends to zero indicating that this is a mode of the vortices themselves, in contrast the normal scissors modes are modes of the condensate whose frequency is perturbed by the presence of vortices (angular momentum). It can be seen directly from the cubic equation that for a condensate in a spherically symmetric potential ($\omega_x = \omega_y = \omega_z$) the ‘tilting’ mode has zero frequency; this arises because vortices have the same energy for any orientation in a spherical cloud and so if the orientation of a vortex is changed with respect to some axis then it will simply remain at the new angle.

Our experiment uses evaporative cooling in a TOP (Time-averaged Orbiting Potential) trap to create a Bose-condensate that contains $\sim 10^5$ atoms of the Rb⁸⁷ isotope in the $|F = 1, m_F = -1\rangle$ state. The oscillation frequencies of atoms in the trap are 61 Hz radially and 172.5 Hz along the z (vertical) direction. After a condensate has been formed we rotate the trapping potential around the z -axis at a frequency of 46.5 Hz, and adiabatically change from a cylindrical trapping potential to a rotating elliptical potential over a period of 200 ms. The potential is rotated for 700 ms at a final ellipticity where $w_x/w_y = 0.95$, before it is ramped back to an axially-symmetric (circular) potential. During this process vortices nucleate at the edge of the condensate and then move towards the centre of the trap [15]. This procedure introduces ten or more vortices into the condensate (Fig. 3).

The trapping potential is tilted by the addition of an oscillating field in the z -direction, in phase with the usual rotating field in the TOP trap. When no vortices are present, tilting the trapping potential excites the scissors mode as described in [14]. This scissors motion is the superposition of two degenerate modes characterized by functions of the form $f(r)xz$ and $f(r)yz$ [16]. The presence of one or more vortices, breaks the degeneracy and leads to modes described by $f(r)z(x \pm iy)$; these are eigenstates of the z -component of orbital angular momentum L_z . A sudden tilt of the trapping potential excites a superposition of these two counter-rotating modes and the resulting scissors oscillation precesses at a rate equal to the frequency splitting between the two modes divided by two [9, 10]. This motion is illustrated in Fig. 4 for a condensate that contains an array of many vortices. In this case the effective rotation frequency of the condensate Ω_0 is much greater than that for a single vortex. A fit to these data gives the frequencies of the upper and lower scissors modes as $f_u = \omega_u/2\pi = 211.8 \pm 2.0$ Hz and $f_l = 156.3 \pm 2.0$ Hz respectively. We find that this corresponds to an average angular momentum per particle of $\langle l_z \rangle = 8.4 \pm 0.4 \hbar$, for a total number of atoms $N = 75\,000$. A vortex at the trap centre contributes \hbar of angular mo-

mentum per particle but off-centre vortices have a smaller contribution [2]. From these measured frequencies and Eq. 1, an effective rotation of $\Omega_0 = 0.495 \pm 0.019$ can be deduced, which implies a tilting mode frequency $\omega_t = 23.5 \pm 0.9$ Hz according to the hydrodynamic theory.

To excite the tilting mode of the vortex array the tilt angle of the trapping potential (relative to the z axis) was driven at a frequency of 61 Hz for two complete cycles of oscillation with an amplitude of 0.07 radians. After excitation, the condensate was held in the trap for a variable amount of time during which the tilting mode evolved at its natural frequency; the condensate was then released and allowed to expand freely for 19 ms before a laser beam was flashed on to record an image of the cloud. (This imaging was destructive.) When no vortices were present in the condensate the response to the driving was an excitation of the normal scissors mode with a small amplitude, as in Fig. 5, because this mode is far from resonance. For a condensate containing vortices, however, there was near-resonant driving of the tilting mode, which resulted in large amplitude oscillations of this mode. The tilting of the vortex array leads to a change in angle of the condensate and the projection of this motion onto the imaging direction gives a signal similar to that of the normal scissors mode but at a much lower frequency. In a random sample of the absorption images the vortex cores line up with the imaging beam: in these images it is possible to see that the vortices are indeed tilting in unison when the mode is excited (see Fig. 6). The tilting mode was excited both along the direction of the imaging beam and in a direction perpendicular to it (Fig. 7). This allowed us to determine the direction of rotation of the tilting mode, and to verify that it rotates in the opposite direction to the initial rotation that creates the vortices (the initial rotation has the same direction as the flow around the vortices). In a Kelvin wave the vortex core has the form of a helix of a particular handedness with respect to the direction of rotation of the vortex; this property also applies to the collective tilting of the vortex array. A sinusoidal fit to the data gives a frequency of 57.7 ± 1.3 Hz for the tilting mode.

The number and position of the vortices varies from shot to shot resulting in slightly different frequencies of the tilting mode so that after a few cycles the observations have more fluctuations (i.e. this method of recording the data has a dephasing analogous to the transverse relaxation in magnetic resonance techniques, and we have not measured the damping of this mode equivalent to longitudinal relaxation time.) The uncertainty produced by variations in the initial conditions could be reduced by taking more data for each evolution time and averaging, as in previous work on the superfluid gyroscope [9].

Using the measured values of f_u , f_l and f_t we find the ratio of frequencies $(f_u - f_l)/(2f_t) = 27.7/57.8 = 0.48 \pm 0.03$ that is not consistent with predictions of the

hydrodynamic theory, shown in Fig. 2 (b). If we assume that the scissors modes have the correct frequencies, then the measured value for the tilting frequency is about 2.5 times the hydrodynamic prediction. We should emphasize, however, that we cannot determine from the frequency measurements whether the scissors modes are more accurately described by the theory than the tilting mode. One way of checking this is to estimate the contributions to the angular momentum from the number of observed vortices and their position within the cloud, and compare this with the value obtained from the scissors mode splitting: a vortex at radius r in a condensate of Thomas-Fermi radius R contributes $N\hbar(1 - r^2/R^2)^{5/2}$ to the total angular momentum. In this way we found a total angular momentum per particle of $8.5 \pm 1.0 \hbar$ for the image in Fig. 3, which is in good agreement with the value calculated from the scissors mode frequency.

If the prediction of the scissors mode frequency is correct, then we need to explain the difference between the predicted and measured values of the tilting mode frequency. Firstly, to eliminate the possibility that we were exciting a higher order Kelvin mode, the tilt angle of the trapping potential was driven at 25 Hz after a vortex array had been nucleated. This driving frequency is close to the hydrodynamic prediction, but no response from the condensate was observed. If we are definitely driving the lowest order mode, the case for which is supported by the straightness of the vortex cores in Fig. 6, then the difference is most likely explained by considering the range of applicability of the hydrodynamic theory; indeed, the theory relies on the fact that it is possible to average physical quantities over domains containing several vortices [13]. For a single vortex a full numerical simulation is currently possible, as shown in Fig. 1 for our experimental conditions. The tilting mode frequency is calculated to be $f_t = 0.27\omega_z$, where ω_z is the axial trapping frequency; the splitting between the scissors modes is calculated to be $0.004\omega_z$, thus for a single vortex $(f_u - f_l)/(2f_t) = 0.09$. This deviates even further from the hydrodynamic prediction that the frequency ratio is about unity than the results for our small vortex array. The hydrodynamic theory only gives an accurate description for arrays that contain a large number of vortices, the vortex arrays in this experiment do not contain a sufficient number of vortices to be in the regime where it is possible to average over domains of vortices and apply the hydrodynamic theory (Fig. 3); increasing the value of experiment in this intermediate region. The hydrodynamic theory should, however, be a better approximation

for larger vortex arrays as in references [6, 7].

In conclusion, this experimental measurement of the frequency and direction of rotation of the collective tilting mode of the vortex array qualitatively supports the predictions made by Chevy and Stringari [13]. The cause of the discrepancy between our measurements and the hydrodynamic predictions could be investigated experimentally with a larger condensate that can contain a higher number of vortices or theoretically by a numerical simulation as in [17]. The experimental method for direct excitation of a vortex (or vortex array) can also be used to study higher-order Kelvin waves.

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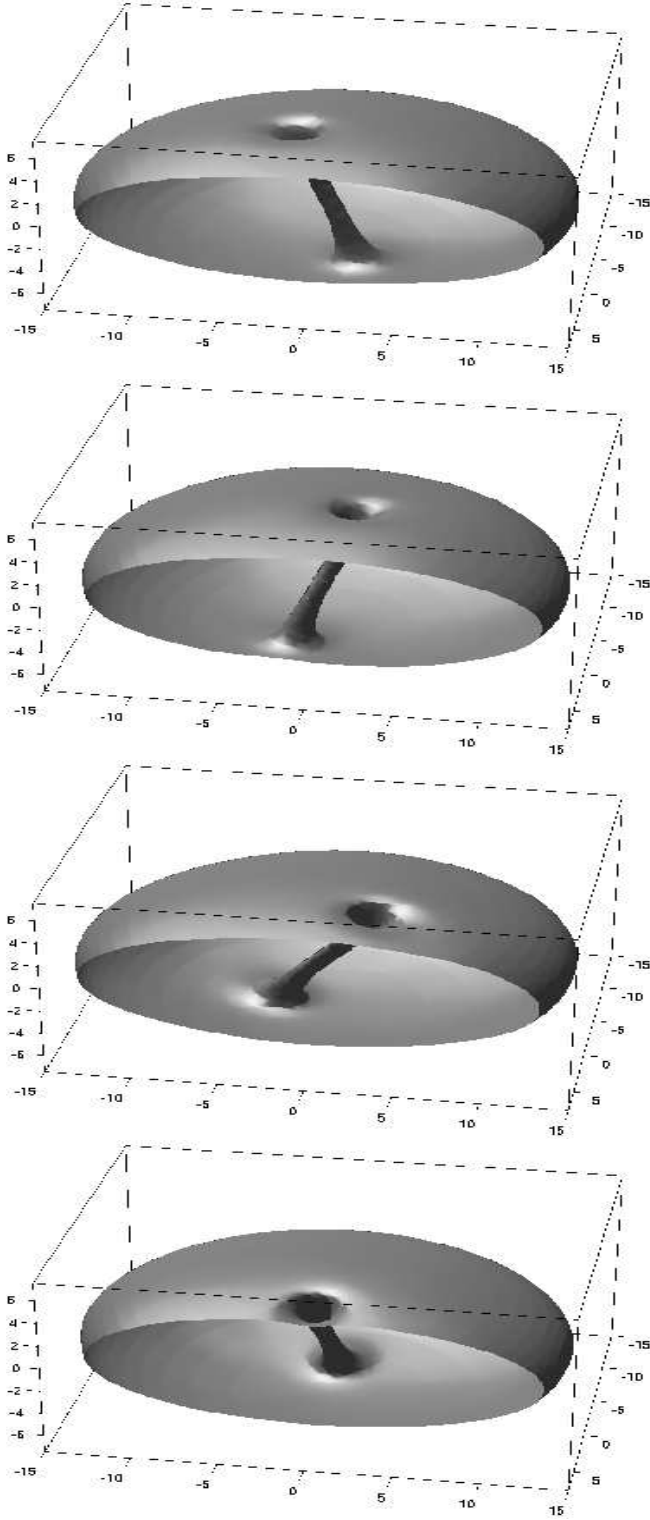


FIG. 1: The lowest-energy Kelvin wave of a single vortex corresponds to a rotation of the tilted vortex line in the opposite direction to the rotation of the condensate. The plots show one complete rotation of the vortex going from top to bottom. In the tilting mode of a vortex array excited in this experiment each vortex synchronously undergoes a similar motion. The plots show surfaces of constant density $|\Psi|^2 = 9 \times 10^{-4}$ of a condensate cloud (wave function normalized to one) from a numerical simulation using the Gross-Pitaevskii equation. The initial state is a single centred vortex (aligned along the vertical axis); this is excited by resonant driving for two vortex precession periods by tilting the confining potential about an axis in the horizontal plane, as in the experiments, with an amplitude of 4° . In this simulation $N = 75000$, $f_\perp = 61$ Hz, $f_z = 172.5$ Hz.

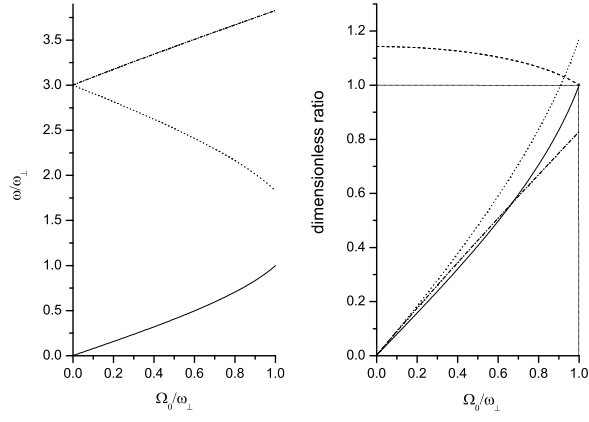


FIG. 2: Calculation of the mode frequencies predicted by Eq. 1 for a trapping potential with $\omega_z/\omega_\perp = \sqrt{8}$. (a) ω_u/ω_\perp (dot-dash line), ω_l/ω_\perp (dotted line), ω_t/ω_\perp (solid line). (b) $\omega_u/\omega_\perp - 3$ (dot-dash line), $3 - \omega_l/\omega_\perp$ (dotted line), ω_t/ω_\perp (solid line). The plot also shows the ratio $(\omega_u - \omega_l)/2\omega_t$ (dashed line).

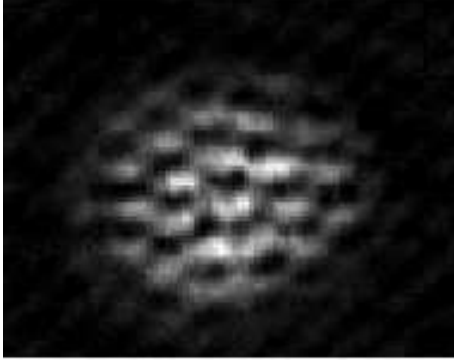


FIG. 3: Absorption image of a vortex array, taken along z axis.

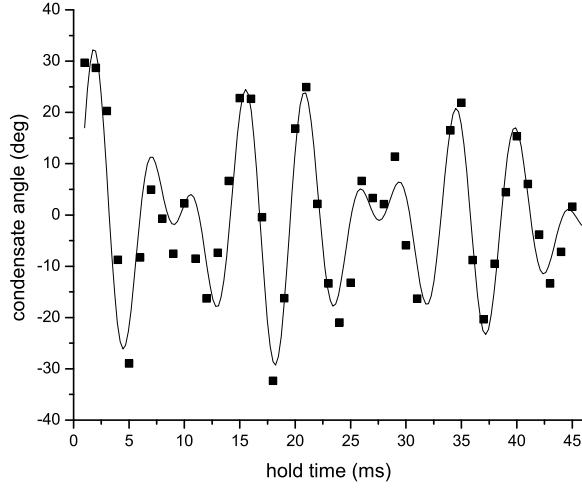


FIG. 4: Exciting the normal scissors mode in the presence of vortices produces a precession that can be used to infer the angular momentum of the vortex array.

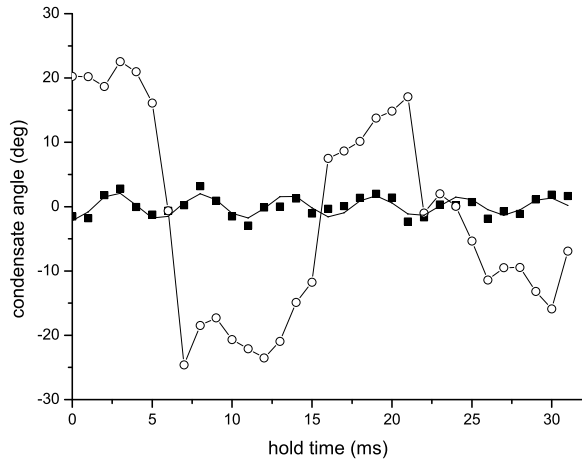


FIG. 5: (black squares) Off resonant excitation of the scissors mode when no vortices are present. (white circles) Resonant driving of the ‘tilting mode’ in the presence of a vortex array.

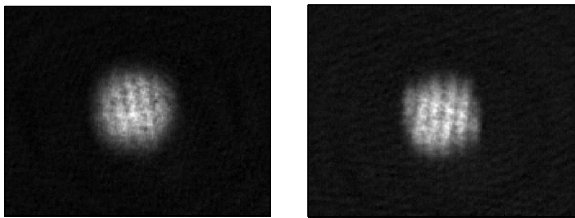


FIG. 6: Absorption images of the vortex lattice, viewed in a radial direction.

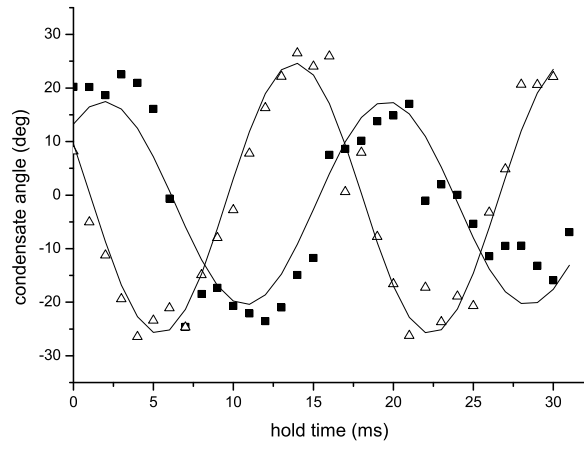


FIG. 7: (black squares) Anomalous mode initially excited perpendicular to probe beam. (white triangles) Anomalous mode initially excited parallel to probe beam.